Zeno of Elea (b. 488 BCE.)

Zeno was an Eleatic philosopher, a native of Elea (Velia) in Italy, son of Teleutagoras, and the favorite disciple of Parmenides. He was born about 488 BCE., and at the age of forty accompanied Parmenides to Athens. He appears to have resided some time at Athens, and is said to have unfolded his doctrines to people like Pericles and Callias for the price of 100 minae. Zeno is said to have taken part in the legislation of Parmenides, to the maintenance of which the citizens of Elea had pledged themselves every year by oath. His love of freedom is shown by the courage with which he exposed his life in order to deliver his native country from a tyrant. Whether he died in the attempt or survived the fall of the tyrant is a point on which the authorities vary. They also state the name of the tyranny differently. Zeno devoted all his energies to explain and develop the philosophical system of Parmenides. We learn from Plato that Zeno was twenty-five years younger than Parmenides, and he wrote his defense of Parmenides as a young man. Because only a few fragments of Zeno's writings have been found, most of what we know of Zeno comes from what Aristotle said about him in *Physics*, Book 6, chapter 9.

Zeno's contribution to Eleatic philosophy is entirely negative. He did not add anything positive to the teachings of Parmenides, but devoted himself to refuting the views of the opponents of Parmenides. Parmenides had taught that the world of sense is an illusion because it consists of motion (or change) and plurality (or multiplicity or the many). True Being is absolutely one; there is in it no plurality. True Being is absolutely static and unchangeable. Common sense says there is both motion and plurality. This is the Pythagorean notion of reality against which Zeno directed his arguments. Zeno showed that the common sense notion of reality leads to consequences at least as paradoxical as his master's.

Paradoxes of Multiplicity and Motion. Zeno's arguments can be classified into two groups. The first group contains paradoxes against multiplicity, and are directed to showing that the 'unlimited' or the continuous, cannot be composed of units however small and however many. There are two principal arguments:

1. If we assume that a line segment is composed of a multiplicity of points, then we can always bisect a line segment, and every bisection leaves us with a line segment that can itself be bisected. Continuing with the bisection process, we never come to a point, a stopping place, so a line cannot be composed of points.

2. The many, the line, must be both limited and unlimited in number of points. It must be limited because it is just as many (points) as it is, no more, and less. It is therefore, a definite number, and a definite number is a finite or limited number. However, the many

must also be unlimited in number, for it is infinitely divisible. Therefore, it's contradictory to suppose a line is composed of a multiplicity of points.

The second group of Zeno's arguments concern motion. They introduce the element of time, and are directed to showing that time is no more a sum of moments than a line is a sum of points. There are four of these arguments:

1. If a thing moves from one point in space to another, it must first traverse half the distance. Before it can do that, it must traverse a half of the half, and so on ad infinitum. It must, therefore, pass through an infinite number of points, and that is impossible in a finite time.

2. In a race in which the tortoise has a head start, the swifter-running Achilles can never overtake the tortoise. Before he comes up to the point at which the tortoise started, the tortoise will have got a little way, and so on ad infinitum.

3. The flying arrow is at rest. At any given moment it is in a space equal to its own length, and therefore is at rest at that moment. So, it's at rest at all moments. The sum of an infinite number of these positions of rest is not a motion.

4. Suppose there are three arrows. Arrow B is at rest. Suppose A moves to the right past B, and C moves to the left past B, at the same rate. Then A will move past C at twice the rate. This doubling would be contradictory if we were to assume that time and space are atomistic. To see the contradiction, consider this position as the chains of atoms pass each other:
A1 A2 A3 ==>
B1 B2 B3
C1 C2 C3 <==
Atom A1 is now lined up with C1, but an instant ago A3 was lined up with C1, and A1 was still two positions from C1. In that one unit of time, A2 must have passed C1 and lined up with C2. How did A2 have time for two different events (namely, passing C1 and lining up with C2) if it had only one unit of time available? It takes time to have an event, doesn't it?

   Both groups of Zeno's arguments, those against multiplicity and those against motion, are variations of one argument that applies equally to space or time. For simplicity, we will consider it only in its spatial sense. Any quantity of space, say the space enclosed within a circle, must either be composed of ultimate indivisible units, or it must be divisible *ad*

infinitum. If it is composed of indivisible units, these must have magnitude, and we are faced with the contradiction of a magnitude which cannot be divided. If it is divisible *ad infinitum*, we are faced with the contradiction of supposing that an infinite number of parts can be added up to make a merely finite sum.

Zeno's point is that since multiplicity and motion contain these contradictions, they cannot be real. Therefore, as Parmenides said, there is only one Being, with no multiplicity in it, and it excludes all motion and change.

Kant's, Hume's, and Hegel's Solutions to Zeno's Paradoxes. According to Kant, these contradictions are immanent in our conceptions of space and time, so space and time are not real. Space and time do not belong to things as they are in themselves, but rather to our way of looking at things. They are forms of our perception. It is our minds which impose space and time upon objects, and not objects which impose space and time upon our minds. Further, Kant drew from these contradictions the conclusion that to comprehend the infinite is beyond the capacity of human reason. He attempted to show that, wherever we try to think the infinite, whether the infinitely large or the infinitely small, we fall into irreconcilable contradictions.

As might be expected, many thinkers have looked for a way out of the paradoxes. [Hume](#) denied the infinite divisibility of space and time, and declared that they are composed of indivisible units having magnitude. But the difficulty that it is impossible to conceive of units having magnitude which are yet indivisible is not satisfactorily explained by Hume.

Hegel believed that any solution which is to be satisfactory must somehow make room for both sides of the contradiction. It will not do to deny one side or the other, to say that one is false and the other true. A true solution is only possible by rising above the level of the two antagonistic principles and taking them both up to the level of a higher conception, in which both opposites are reconciled. Hegel regarded Zeno's paradoxes as examples of the essential contradictory character of reason. All thought, all reason, for Hegel, contains immanent contradictions which it first posits and then reconciles in a higher unity, and this particular contradiction of infinite divisibility is reconciled in the higher notion of *quantity*. The notion of quantity contains two factors, namely the *one* and the *many*. Quantity means precisely a many in one, or a one in many. If, for example, we consider a quantity of anything, say a heap of wheat, this is, in the first place, one; it is one whole. Secondly, it is many, for it is composed of many parts. As one it is continuous; as many it is discrete. Now the true notion of quantity is not one, apart form many, nor many apart from one. It is the synthesis of both. It is a many *in* one. The

antinomy we are considering arises from considering one side of the truth in a false abstraction from the other. To conceive unity as not being in itself multiplicity, or multiplicity as not being unity, is a false abstraction. The thought of the one involves the thought of the many, and the thought of the many involves the thought of the one. You cannot have a many without a one, any more than you can have one end of a stick without the other.

Now, if we consider anything which is quantitatively measured, such as a straight line, we may consider it, in the first place, as one. In that case it is a continuous divisible unit. Next we may regard it as many, in which case it falls into parts. Now each of these parts may again be regarded as one, and as such is an indivisible unit; and again each part may be regarded as many, in which case it falls into further parts; and this alternating process may go on for ever. This is the view of the matter which gives rise to Zeno's contradictions. But it is a false view. It involves the false abstraction of first regarding the many as something that has reality apart from the one, and then regarding the one as something that has reality apart from the many. If you persist in saying that the line is simply one and not many, then there arises the theory of indivisible units. If you persist in saying it is simply many and not one, then it is divisible *ad infinitum*. But the truth is that it is neither simply many nor simply one; it is a many *in* one, that is, it is a *quantity*. Both sides of the contradiction are, therefore, in one sense true, for each is a factor of the truth. But both sides are also false, if and in so far as, each sets itself up as the whole truth.

The Contemporary Solution to Zeno's Paradoxes. Kant's, Hume's and Hegel's solutions to the paradoxes have been very stimulating to subsequent thinkers, but ultimately have not been accepted. There is now general agreement among mathematicians, physicists and philosophers of science on what revisions are necessary in order to escape the contradictions discovered by Zeno's fruitful paradoxes. The concepts of space, time, and motion have to be radically changed, and so do the mathematical concepts of line, number, measure, and sum of a series. Zeno's integers have to be replaced by the contemporary notion of real numbers. The new one-dimensional continuum, the standard model of the real numbers under their natural (less-than) order, is a radically different line than what Zeno was imagining. The new line is now the basis for the scientist's notion of distance in space and duration through time. The line is no longer a sum of points, as Zeno supposed, but a set-theoretic union of a non-denumerably infinite number of unit sets of points. Only in this way can we make sense of higher dimensional objects such as the one-dimensional line and the two-dimensional plane being composed of zero-dimensional points, for, as Zeno knew, a simple sum of even an infinity of zeros would never total more than zero. The points in a line are so densely packed that no point is next to any other point. Between any two there is a third, all the way 'down.' The infinity of points in the line is much larger than any infinity Zeno could have imagined. The non-denumerable infinity of real numbers (and thus of points in space and of events in time) is much larger than the merely denumerable infinity of integers. Also, the sum of an infinite

series of numbers can now have a finite sum, unlike in Zeno's day. With all these changes, mathematicians and scientists can say that all of Zeno's arguments are based on what are now false assumptions and that no Zeno-like paradoxes can be created within modern math and science. Achilles catches his tortoise, the flying arrow moves, and it's possible to go to an infinite number of places in a finite time, without contradiction.

No single person can be credited with having shown how to solve Zeno's paradoxes. There have been essential contributions starting from the calculus of Newton and Leibniz and ending at the beginning of the twentieth century with the mathematical advances of Cauchy, Weierstrass, Dedekind, Cantor, Einstein, and Lebesque. Philosophically, the single greatest contribution was to replace a reliance on what humans can imagine with a reliance on creating logically consistent mathematical concepts that can promote quantitative science.

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